Tests E&M II 6-10-2000

a) $V_B = \dfrac{R}{R + \frac{1}{j\omega C}} V_0 = \dfrac{j\omega RC}{1 + j\omega RC} V_0$

b) $V_B = \dfrac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} \cos(\omega t + \varphi)$ met $\varphi = \frac{\pi}{2} - \arctan(\omega RC)$

c) $\dfrac{1}{Z} = \dfrac{1}{R + j\omega L} + \dfrac{1}{R + \frac{1}{j\omega C}} = \dfrac{1}{R + j\omega L} + \dfrac{j\omega C}{1 + j\omega RC}$

$= \dfrac{1 + j\omega RC + j\omega RC - \omega^2 LC}{R - \omega^2 LRC + j\omega L + j\omega R^2 C} = \dfrac{(1-\omega^2 LC) + 2j\omega RC}{R(1-\omega^2LC) + j\omega(L + R^2C)}$

$\therefore I = \dfrac{V_0}{Z} = \dfrac{(1 - \omega^2 LC + 2\omega RC) V_0}{R(1-\omega^2LC) + j\omega(L+R^2C)}$

) $\vec{E}_1 = E\hat{y} \longrightarrow \vec{E}_1^{\parallel} = -\frac{1}{2}E\hat{x} + \frac{1}{2}E\hat{y} \rightarrow E_1^{\parallel} = \sqrt{\frac{1}{4}+\frac{1}{4}}\,E = \frac{1}{2}\sqrt{2}\,E$

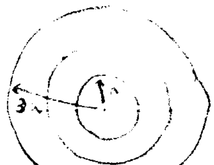
$\vec{E}_1^{\perp} = \frac{1}{2}E\hat{x} + \frac{1}{2}E\hat{y}$

$E_2^{\parallel} = E_1^{\parallel} \longrightarrow \vec{E}_2^{\parallel} = -\frac{1}{2}E\hat{x} + \frac{1}{2}E\hat{y} \rightarrow |E_2^{\parallel}| = E$

$\varepsilon_{r_2} E_2^{\perp} = \varepsilon_{r_1} E_1^{\perp} \rightarrow 2E_2^{\perp} = E_1^{\perp} \rightarrow \vec{E}_2^{\perp} = \frac{1}{4}E\hat{x} + \frac{1}{2}E\hat{y} \rightarrow |E_2^{\perp}| = \frac{\sqrt{5}}{4}E$

$\vec{E}_2 = \vec{E}_2^{\parallel} + \vec{E}_2^{\perp} = -\frac{1}{4}E\hat{x} + \frac{3}{4}E\hat{y} \rightarrow |E_2| = \sqrt{\frac{5}{8}}\,E$

) $a < r < 2a \qquad E = \dfrac{\lambda}{2\pi\varepsilon_0\varepsilon_r r} = \dfrac{\lambda}{6\pi\varepsilon_0 r}$



$2a < r < 4a \qquad E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$V = -\int_a^{4a} \vec{E}\cdot d\vec{r} = -\int_a^{2a} \dfrac{\lambda}{6\pi\varepsilon_0 r}\,dr - \int_{2a}^{4a} \dfrac{\lambda}{2\pi\varepsilon_0 r}\,dr = \dfrac{-\lambda}{6\pi\varepsilon_0}\left\{\ln 2 + 3\ln 2\right\} = \dfrac{-2\lambda \ln 2}{3\pi\varepsilon_0}$

$C = \dfrac{Q}{V} \rightarrow \dfrac{C}{l} = \dfrac{\lambda}{|V|} = + \dfrac{3\pi\varepsilon_0}{2 \ln 2}$

b) $\sigma_p = P_n = \varepsilon_0(\varepsilon_r - 1)E = 2\varepsilon_0 E \quad \sigma_p = \dfrac{\lambda}{6\pi a} \rightarrow \lambda_p = 2\pi(2a)\sigma_p = \frac{2}{3}\lambda$ {half elim...}

$E(2a) = \dfrac{\lambda}{12\pi \ldots}$